MATH 5C - TEST 2 sample
(Chapter 14)

Note: This is a little longer than the test

(1) Find and sketch the domain of f(x,y) = $\sqrt{2x^2 - y}$. Evaluate f(3,2).

(4 points)

(2) Use the chain rule to find $\frac{\partial w}{\partial t}$ given w= f(x, y, z) = x^2 cos(3y)+ ln(z) ;

x = 7t – 4s, y=5st, z= t cos(s). (No need to simplify answer all in terms of s, t .) Show the chain rule formula you used.

(5 points)

(3) Find all critical points of f(x,y) =$x^2 - x^2y + 2y^2$ and classify each as yielding local max., local min., or saddle points. Show how you arrive at your conclusions. You do not need to find the functional values at the critical points.

(10 points)

(4) Find the equation of the plane tangent to the surface $z = x^2 e^{2y}$ at the point (3, 0, 9).
(10 points)

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(5) For the function $f(x,y) = \dfrac{x^3 y}{3x^6 + y^2}$, (SHOW WORK) (8 points)

(a) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along any straight line y = mx. ______________________

(b) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the parabola $y=x^2$.______________________

(c) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the curve $y = x^3$.______________________

(d) What can be said about $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?____________________________

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(6) Given $z = y^2 - 5xy$. If x changes from 3 to 3.1 and y changes from 5 to 4.8, find and compare the values of Δz and dz.

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(8 points)

(7) The temperature at a point (x, y) on a metal plate in the xy-plane is $T(x,y) = 20-4x^2 - y^2$ degrees Celsius where x and y are measured in centimeters.

(15 points)

(a) Find the rate of change of the temperature at (2,-3) as we move towards (5, -2). Is the temperature getting warmer or colder? (Show correct units in answer.)

(b) In what direction from (2, -3) does the temperature increase most rapidly? What is the rate of increase?

(c) Find $T_y(-1,1)$. Describe the physical meaning of this in terms of temperature on the metal plate.

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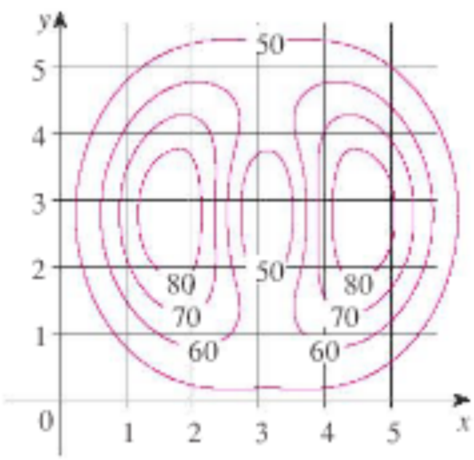
(8) Given the following level curves for an unknown function f(x,y),



Estimate the following. Show work on b,c,d. (8 points)

(a) f(1,2) ________________

(b) $\left.\dfrac{\partial f}{\partial x}\right|_{(1,2)}$ ________________

(c) $f_y(4,4)$________________

(d) $D_{\vec{u}} f(1,4)$ where $\vec{u} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$________

(9) Match the following functions with their graphs:

(8 points)

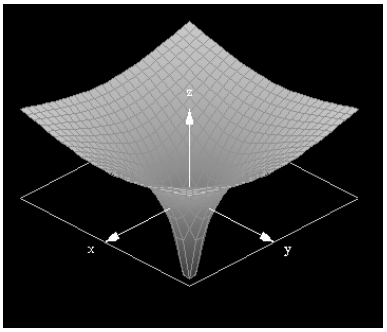
a) $f(x,y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ __________

(b) $f(x,y) = \cos(e^x+e^y)$ __________

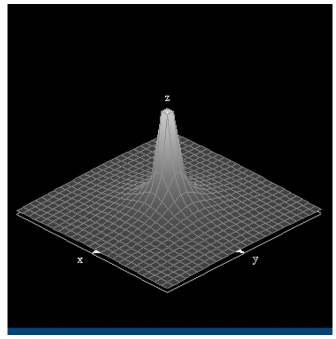
c) $f(x,y) = \ln(x^2+y^2)$ __________

(d) $f(x,y) = \cos(xy)$______________

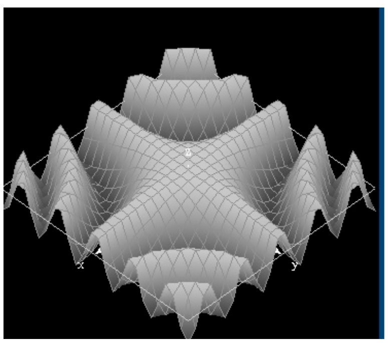
A



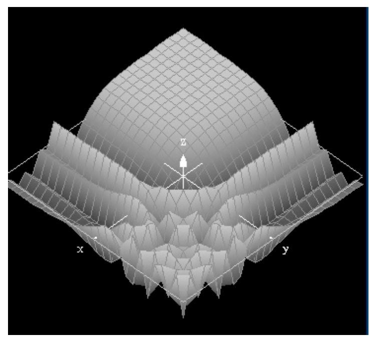
B



C



D



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(10) Find the absolute extreme values of $f(x,y) = x^2 + y^2 - 2x - 4y$ on the region bounded by $y=x$, $y=3$, and $x=0$. Show all work. In particular, SHOW ALL POINTS WHICH YOU CONSIDERED AS POSSIBILITIES FOR YIELDING EXTREME VALUES.

(11) Use the method of Lagrange multipliers to show that the rectangle of largest area that can be inscribed in a circle is a square. Give the maximum rectangle area in terms of circle radius.

Show all points which you considered as possibilities for yielding extreme values.

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(12)

4. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in the following table.

Duration (hours)

Wind speed (knots)

v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

(a) Find f(40,30) and clearly interpret the physical meaning with units.

(b) Estimate $\frac{\partial f}{\partial v}(40,30)$ and $f_t(40,30)$. Only one estimate needed for each. Interpret the physical meaning. Give proper units. Show work.

(13) Given that z implicitly represents a function of x and y in the following equation, $xz = z^2 \ln y$ find $\frac{\partial z}{\partial x}$

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(14) Use differentials or a linear approximation to approximate the value of $\sqrt{26} + \sqrt[3]{63}$ without using your calculator. (You can use your calculator to check your result).